Momentum Balance - An Example Application

Consider steady flow of hydraulic fluid (oil) in a pipe constriction that is part of a hydraulic apparatus. Determine the force necessary to keep the pipe constriction in place.

\[ Q = 0.082 \text{ m}^3/\text{sec} \]

\[ p_1 = 323 \text{ kPa} \quad p_2 = 40 \text{ kPa} \quad \rho = 0.8 \text{ gm/cm}^3 = 800 \text{ kg/m}^3 \]

\[ D_1 = 0.30 \text{ m} \quad D_2 = 0.07 \text{ m} \]

Steady flow => 1. that \( Q \) is constant
2. that there is no accumulation of momentum
3. that all forces are constant in time (since \( a = 0 \))

\[ \sum \text{ Forces} = \rho_1 A_1 - \rho_2 A_2 - F \]

\( x \)-direction taken as \( @ \) to the right

\[ = \rho_1 (V_1^2 A_1 - V_2^2 A_2) = \rho_2 Q (V_2 - V_1) \]

\[ F = \rho_2 Q (V_2 - V_1) \]

\[ A_1 = \pi (0.15 \text{ m})^2 = 0.0707 \text{ m}^2 \]
\[ A_2 = \pi (0.07 \text{ m})^2 = 0.0038 \text{ m}^2 \]

\[ (\rho_1 A_1 - \rho_2 A_2) = (22,678 - 1842 \text{ N}) = 20,836 \text{ N} \]

\[ V_1 = \frac{Q}{A_1} = \frac{0.082 \text{ m}^3/\text{sec}}{0.0707 \text{ m}^2} = 1.16 \text{ m/sec} \]

\[ p_0 Q (V_2 - V_1) = (800 \frac{\text{kg}}{\text{m}^3})(0.082 \text{ m}^3/\text{sec})(21.6 - 1.16) \]

\[ V_2 = \frac{Q}{A_2} = \frac{0.082 \text{ m}^3/\text{sec}}{0.0038 \text{ m}^2} = 21.4 \text{ m/sec} \]

\[ F = (\rho_1 A_1 - \rho_2 A_2) + (\rho_2 Q (V_2 - V_1)) = 22,678 + 1842 \text{ N} = 24,521 \text{ N} \]