Solution of Problem Set #3, 2002

1) 
   a) $A \rightarrow B$; $h(\text{sand}) > h(\text{clay})$ (See Chapter 2 p.16 of lecture note)
   b) $B \rightarrow A$; water surface tension will decrease as temperatures goes up, so water potential will be less negative ($\rho gh = 2\sigma \cos \theta / R$). We assume same radius of pore size.
   c) No flow. $h$’s are the same.
   d) No flow. Osmotic potential affects water flow only when there is a semi-permeable membrane.
   e) $B \rightarrow A$

2) 
   a) 
   \[ q = -K \frac{\Delta H}{\Delta x} = (-80 \text{ cm/day}) \left( \frac{60 \text{ cm}}{50 \text{ cm}} \right) = -96 \text{ cm/day} \] (negative = flow downward)
   \[ Q = qAt = (-96 \text{ cm/day}) \left( 10\text{cm}^2 \right) \left( 5\text{hours} \right) \left( \frac{1\text{day}}{24\text{hours}} \right) = -200\text{cm}^3 \]

   \[ q = -K \frac{\Delta H}{\Delta x} = (-80 \text{ cm/day}) \left( \frac{-10 \text{ cm}}{50 \text{ cm}} \right) = 16 \text{ cm/day} \]

   b) 
   \[ q = -K \frac{\Delta H}{\Delta x} = (-80 \text{ cm/day}) \left( \frac{-10 \text{ cm}}{50 \text{ cm}} \right) = 16 \text{ cm/day} \]
3) 

<table>
<thead>
<tr>
<th></th>
<th>Point A</th>
<th>Point B</th>
<th>Point C</th>
</tr>
</thead>
<tbody>
<tr>
<td>z (cm)</td>
<td>0</td>
<td>30</td>
<td>40</td>
</tr>
<tr>
<td>h (cm)</td>
<td>50</td>
<td>10 (from the opening end)</td>
<td>0 (open to the atm)</td>
</tr>
<tr>
<td>H (cm)</td>
<td>50</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
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4) 
Set the reference at point E

At point C, 
h = -30 cm  
z = 42 cm  
\( H_C = -30 + 42 = 12 \) cm

At point E, 
h = 0 cm  
z = 0 cm  
\( H_E = 0 \) cm

At point A, 
h = 0 cm  
z = 30 cm  
\( H_A = 30 \) cm

Assume uniform water content and constant saturated hydraulic conductivity \( (K_s) \) throughout the column, thus, uniform gradient is along the column

\[
\text{gradient} = \frac{H_E - H_C}{\text{Distance between E and C}} = \frac{0 - 12}{(0-42)} = 0.286
\]

\( q = -K_s \left( \frac{\Delta H}{\Delta x} \right) = -0.286K \) (the negative value indicates water flow downward)

at point B
water flow upward from point A to point B, in order to correct the flow direction, so \( q = +0.286K \) and \( (\Delta H/\Delta x) = -0.286 \)

\[
\text{gradient} = \frac{H_B - H_A}{\text{Distance between B and A}} = \frac{H_B - 30}{42 - 30} = 0.286
\]

\( H_B = 26.57 \) cm
z = 42 cm
h = 26.57 - 42 = -15.43 cm

for the distance between B and C
water flow from point B to C (from left to right)  
so \( q = +0.286K \) and \( (\Delta H/\Delta x) = -0.286 \)
- 0.286 = \( \frac{(H_C - H_B)}{BC} \) = \( \frac{(12 - 26.57)}{BC} \)

BC = 50.94 cm

at point D

water flow downward from point D to point E,

so \( q = -0.286K \) and \( \frac{\Delta H}{\Delta x} = +ve \)

\( 0.286 = \frac{(0 - H_d)}{(0 - 30)} \)

\( H_d = 8.58 \)

\( z = 30 \)

\( h = 8.58 - 30 = -21.42 \)

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<th>Point E</th>
</tr>
</thead>
<tbody>
<tr>
<td>z</td>
<td>30</td>
<td>42</td>
<td>42</td>
<td>30</td>
</tr>
<tr>
<td>h</td>
<td>0</td>
<td>-15.43</td>
<td>-30</td>
<td>-21.42</td>
</tr>
<tr>
<td>H</td>
<td>30</td>
<td>26.57</td>
<td>12</td>
<td>8.58</td>
</tr>
</tbody>
</table>

Water potential

Given that \( K_1 = 6K_2 \),

steady state, therefore, flux density through soil 1 is same as soil 2 \( \rightarrow q_1 = q_2 \)

\( q_1 = -K_1 \left( \frac{\Delta H}{\Delta z} \right) \)

\( = -K_1 \left( 90 - (h_B + 20) \right) / 60 \)

\( = -K_1 \left( 70 - h_B \right) / 60 \)

\( q_2 = -K_2 \left( \frac{\Delta H}{\Delta z} \right) \)

\( = -K_2 \left( (20 + h_B) - 0 \right) / 20 \)

\( = -K_2 \left( 20 + h_B \right) / 20 \)
\( q_1 = q_2 \) and \( K_1 = 6 \ K_2 \)

Thus,

\[-K_1 (70 - h_B) / 60 = -K_2 (20 + h_B) / 20\]
\[6 (70 - h_B) / 60 = (20 + h_B) / 20\]

\( \therefore h_B = 40 \) cm

\( H_B = h_B + z = 40 + 20 = 60 \) cm

\( q = -K_2 (20 + 40) / 20 \)
\[= -1.2 \ \text{cm/day (60/20)} \]
\[= -3.6 \ \text{cm/day (negative mean water flow downward)} \]

b)

\( q_1 = -K_1 (\Delta H / \Delta z) \)
\[= -K_1 (h_B + 60) / 60 \]

\( q_2 = -K_2 (\Delta H / \Delta z) \)
\[= -K_2 \{90 - (60+h_B)\} / 20 \]
\[= -K_2 (30 - h_B) / 20 \]

\( q_1 = q_2 \) and \( K_1 = 6 \ K_2 \)

Thus,

\[-K_1 (h_B + 60) / 60 - K_2 (30 - h_B) / 20\]
\[6 (h_B + 60) / 60 = (30 - h_B) / 20 \]

\( \therefore h_B = -30 \) cm

\( H_B = h_B + z = -30 + 60 = 30 \) cm

\( q = -K_2 \{30 - (-30)\} / 20 \)
\[= -1.2 (60/20) \]
\[= -3.6 \ \text{cm/day} \)