Momentum Balance – An Example Application

Consider steady flow of hydraulic fluid (oil) in a pipe constriction that is part of a hydraulic apparatus. Determine the force necessary to keep the pipe constriction in place.

$p_1 = 323$ kPa $p_2 = 40$ kPa $\rho_{oil} = 0.8$ gm/cm³ $= 800$ kg/m³

$D_1 = 0.30$ m $D_2 = 0.07$ m

Steady flow ⇒ 1. that Q is constant.
2. that there is no accumulation of momentum.
3. that all forces are constant in time (same as # 2).

Σ Forces $= p_1 A_1 - p_2 A_2 - F$
(x-direction taken as ⊕ to the right)
$= -\rho_o (V_2^2 A_2 - V_1^2 A_1) = \rho_o Q (V_1 - V_2)$
} Momentum Balance

$\therefore \quad F = (p_1 A_1 - p_2 A_2) - \rho_o Q (V_1 - V_2)$

$A_1 = \pi (0.15\,m)^2 = 0.0225\pi\ m^2 = 0.0707\ m^2$

$A_2 = \pi (0.035\,m)^2 = 0.0012\pi\ m^2 = 0.0038\ m^2$

$(p_1 A_1 - p_2 A_2) = (22\,832 - 154\ N) = 22\,678\ N$

$V_1 = \frac{Q}{A_1} = \frac{0.082\ m^3/s}{0.0707\ m^2} = 1.16\ m/s$

$V_2 = \frac{Q}{A_2} = \frac{0.082}{0.0038} = 21.6\ m/s$

} $\rho_o Q (V_2 - V_1) = (800\ \frac{kg}{m^3})(0.082\ \frac{m^3}{s})(21.6 - 1.16)$
$= 1343\ N$

$F = (p_1 A_1 - p_2 A_2) + \rho_o Q (V_2 - V_1) = 22678 + 1343\ N = \underline{\underline{24021\ N}}$