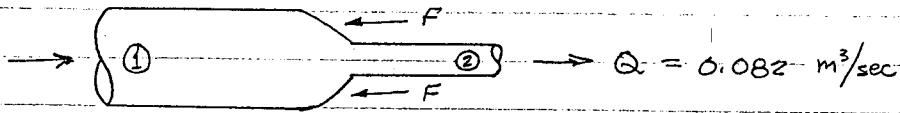


Momentum Balance - An Example Application

Consider steady flow of hydraulic fluid (oil) in a pipe constriction that is part of a hydraulic apparatus. Determine the force necessary to keep the pipe constriction in place.



$$p_1 = 323 \text{ kPa}$$

$$D_1 = 0.30 \text{ m}$$

$$p_2 = 40 \text{ kPa}$$

$$D_2 = 0.07 \text{ m}$$

$$\rho_{\text{oil}} = 0.8 \text{ gm/cm}^3 = 800 \text{ kg/m}^3$$

Steady flow \Rightarrow 1. that Q is constant.

2. that there is no accumulation of momentum.

3. that all forces are constant in time (same as #2).

$$\left. \begin{aligned} \Sigma \text{ Forces} &= -p_1 A_1 - p_2 A_2 - F \\ \text{(x-direction taken as } \oplus \text{ to the right)} \end{aligned} \right\} \text{Momentum Balance}$$

$$= -\rho_0 (v_2^2 A_2 - v_1^2 A_1) = \rho_0 Q (v_1 - v_2)$$

$$\therefore F = (p_1 A_1 - p_2 A_2) - \rho_0 Q (v_1 - v_2)$$

$$A_1 = \pi (0.15 \text{ m})^2 = 0.0225\pi \text{ m}^2 = 0.0707 \text{ m}^2$$

$$A_2 = \pi (0.035 \text{ m})^2 = 0.0012\pi \text{ m}^2 = 0.0038 \text{ m}^2$$

$$(p_1 A_1 - p_2 A_2) = (22832 - 154 \text{ N}) = 22678 \text{ N}$$

$$v_1 = \frac{Q}{A_1} = \frac{0.082 \text{ m}^3/\text{s}}{0.0707 \text{ m}^2} = 1.16 \text{ m/s}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.082}{0.0038} = 21.6 \text{ m/s}$$

$$\left. \begin{aligned} \rho_0 Q (v_2 - v_1) &= (800 \frac{\text{kg}}{\text{m}^3}) (0.082 \frac{\text{m}^3}{\text{s}}) (21.6 - 1.16) \\ &= 1343 \text{ N} \end{aligned} \right\}$$

$$F = (p_1 A_1 - p_2 A_2) + \rho_0 Q (v_2 - v_1) = 22678 + 1343 \text{ N} = \underline{\underline{24021 \text{ N}}}$$