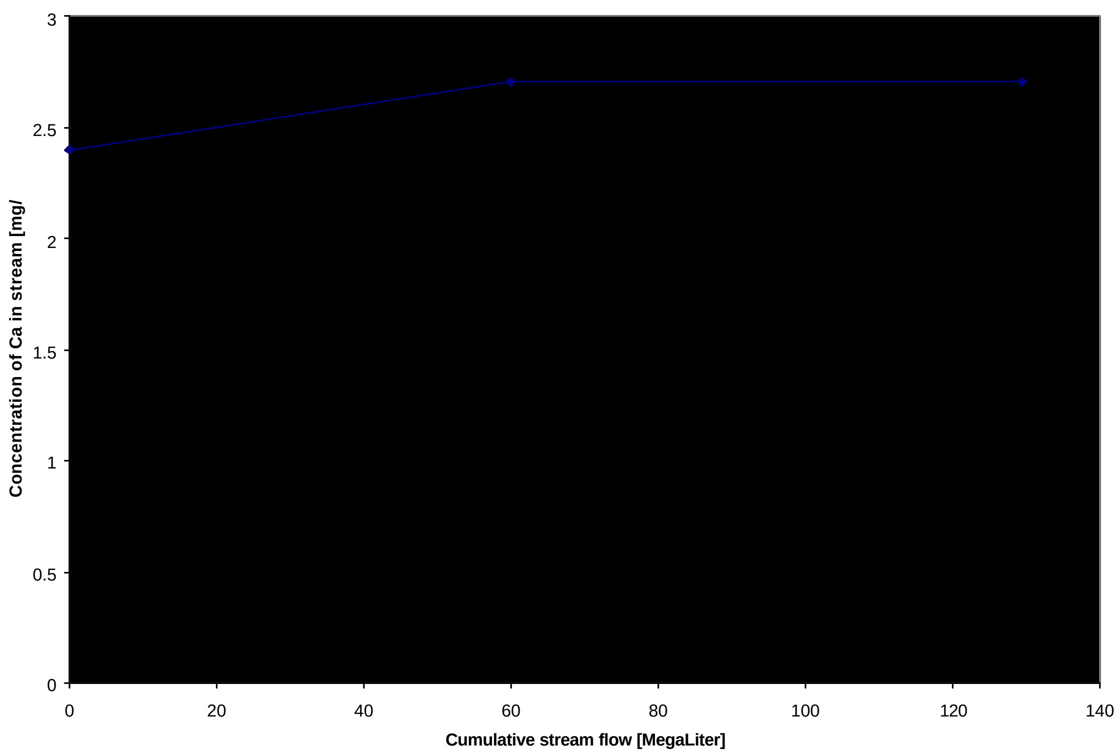
TOTAL **FINAL EXAM**

100 points *ERS 100 – Fall 1999* *Name:*

20 points **1a)**

In a 150 hectare lake, 5 m deep, an initial concentration of 2.4 mg/l of Calcium was measured at the beginning of the summer. The lake contains predominantly $CaCO_3$ water. During the summer, no water was added to the lake, but a small stream out of the lake and evaporation created a drop in water level. The stream had a constant flowrate during the summer of approximately 10 l/s. Researchers estimated that at the end of the summer, an amount of 1.2 Mg (Mega gram) of $CaCO_3$ (calcite) had precipitated in the lake. The surface area of the lake after the level drop did not change. A graduate student created a graph showing the concentration change of the outflowing stream water against the cumulative stream flow out of the lake. What is the evaporation from the lake [mm/day]? Molecular weight of Ca = 40, O = 16, C = 14. Research period is from May 1st until October 1st and contains 150 days (each month contains 30 days). Mega is 10^6



Hint 1: What is the initial mass of calcium in the lake [kg]?
Hint 2: How much calcium [kg] was transported out of the lake through the stream during the year?
Hint 3: How much Ca is left in solution at the end of the summer?

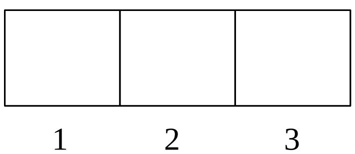
5 points **1b)** At approximately which date during the summer did calcite precipitation first occur?

5 points **1c)** How much did the lake level drop over the summer [mm]?

10 points **1d)** If the lake would have had both a sufficient supply of CO_3^{2-} and SO_4^{2-}, would you expect to find mainly $CaCO_3$ or $CaSO_4$ deposits? Show why…

2)

Three farmers are all farming their adjacent 160 acres of land [1/4th mile by 1/4th mile](see schematic). Because of a drought in 1987, Farmer 1 and Farmer 3 are both running their pumps, which are installed exactly in the center of their fields. Farmer 2 decided not to grow any crops and did not apply any water to his field. In 1988, Farmer 2 installed a groundwater pump too, also exactly in the middle of his field, but he did not grow a crop. The groundwater wells are installed in an unconfined aquifer, 50 m deep (from soil surface to top of the aquiclude). Before pumping, in 1986, the average regional groundwater table was measured at a depth of 2 meter below soil surface. All farmers know how to manage their pumps economically efficient. Since these three farmers know each other, they do not want to affect each other's pumping wells. Pump radii are r_1 = 4 in, r_2 = 4 in, r_3 = 2 in respectively. Saturated hydraulic conductivity is 5 ft/day.



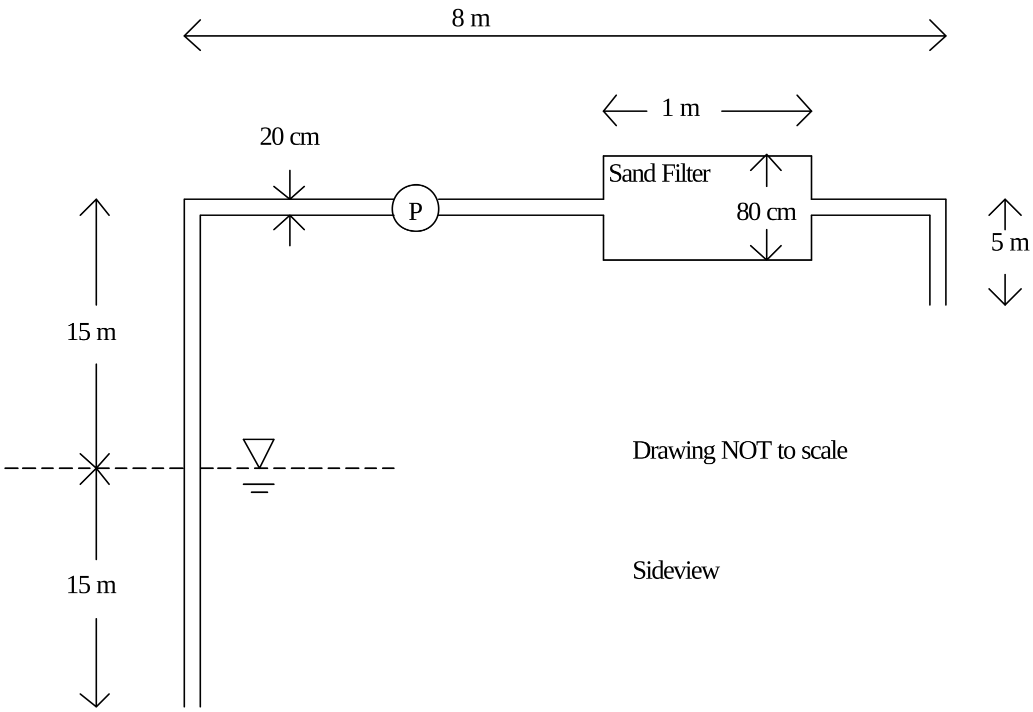
10 points **a)** What are the maximum pumping rates [gpm] for farmer 1 and 3 in 1987, assuming that all water applied to the field was used by the crop?

10 points **b)** What was the daily evapotranspiration [mm/day]?

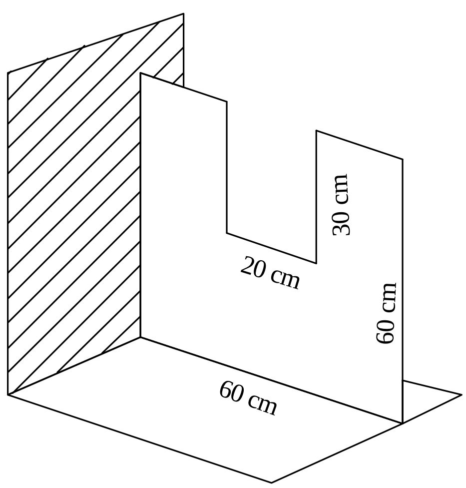
10 points **c)** Farmer 2 uses center pivot sprinkler irrigation (Irrigation is applied on a circle that fits within the square field size). What is the pumping rate for farmer 2 in 1988 if 100% of the pumped water (which is applied to the field) recharges to the groundwater?

10 points **d)** Would it be possible to use the equations on page 171 (Grismer book) for a situation where a crop uses 90% of the applied water (in other words, when recharge is only 10% of the pumping rate). Explain.

10 points **3a)** One of the pumps from question 2 is used to pump water up from 30 m deep. The radius of the pipe used to transport water to the field is 10 cm. Before water is released in an open canal, water is first filtered through a pressurized sand filter (radius is 40 cm) with a hydraulic conductivity of 5 cm/hr. The flowrate is 1200 m^3/day. What is the pump power needed to keep this flowrate if the pump has an efficiency of 75%? Assume that entry-exit losses, friction losses and elbow losses are negligible.



5 points **3b)** Half of the water (600 m^3/day) is pumped into an open canal (n = 0.012). A small rectangular weir is installed as shown in the figure below. What is the critical depth for this weir?



5 points **3c)** What is the slope of the canal when the water height is at the critical depth?

SOLUTIONS FINAL
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1a)

Initial volume
The initial mass of calcium in the lake can be calculated based on the concentration of calcium and the volume of the lake:

Volume of lake = 150 ha * 5 m = 150 * 10,000 * 5 = $7.5 * 10^6$ m^3 = $7.5 * 10^9$ liter
Initial concentration of calcium = 2.4 mg/L
Initial mass of calcium = $2.4 * 10^{-3} * 7.5 * 10^9 = 18 * 10^6$ g Ca = 18,000 kg Ca

Concentration of calcium when ppt occurs

The solubility constant for the reaction below is given in table 10.13

$CaCO_3$ (s) ⇔ Ca^{2+} (aq) + CO_3^{2-} (aq)

$K_{SO} = 4.57 * 10^{-9}$

K_{SO} = [Ca]*[CO_3]

[Ca] = $6.76 * 10^{-5}$ moles/liter

[Ca] = $6.75 * 10^{-5}$ moles/liter * 40 g/mole = $2.70 * 10^{-3}$ g/L = 2.7 mg/L

Weight of Calcium in $CaCO_3$ precipitate = ratio of calcium MW over $CaCO_3$ MW
= 40/102 * 1200 kg = 471 kg

Volume leaving lake during summer in stream

Total volume of water leaving the lake during the summer is
10 l/s * 150 days * 24 hr/day * 3600 s/hr = $129.6 * 10^6$ liters = 129.6 Mliter

The mass of calcium taken out of the lake can be determined by the area under the graph (similar to the hydrographs seen before).

Mass taken out of the lake (in mg)

½ * (2.7-2.4) * ($60*10^6$) + 2.4 * $60*10^6$ + 2.7 * ($129.6*10^6$ - $50*10^6$) =

341,042,130 mg → 341.0 kg Ca removed from the lake.

Volume left in lake at end of summer

18,000 – 341 kg outflow – 471 kg ppt = 17,188 kg calcium left in solution in the lake

At a concentration of 2.7 mg/L, this means that there are

$$17.188 * 10^3 / 2.7 * 10^{-3} = 6.36 * 10^9 \text{ Liters left in the lake}$$

Water balance

Initially in the lake = $7.9 * 10^9$ liters
Leaving through the streamflow = $0.13 * 10^9$ liters
Left over in lake = $6.36 * 10^9$ liters.

Evaporation = $(7.9 – 0.13 – 6.36) * 10^9 = 1.01 * 10^9$ liters = $1.01 * 10^6$ m^3

Divided over the area of the lake (150 ha) gives then a total evaporation of 0.676 m for 150 days

E = <u>4.51 mm/day</u>

1b)

From the graph it an be seen that the maximum concentration of Ca occurs after 60 ML outflow of the river, with a stream flow of 10 l/s.

$60*10^6 / 10 = 60*10^5$ seconds, or 69 days (or 3 months and 9 days → <u>June 9</u>)

1c)

Initial level was 5 m
Streamflow created a drop of 86.4 mm
Evaporation created a drop of 837 mm

Total drop is thus <u>837 mm</u>, a bit less than 1 meter

1d)

$K_{SO}\{CaCO_3\} << K_{SO}\{CaSO_4\}$
Thus, solubility of calcium carbonate is lower than the solubility of calcium sulfate.
When both carbonates and sulfates are available in unlimited amounts, the concentration

of calcium will be at the level of the solubility of $CaCO_3$. More precipitation of $CaCO_3$ will be found than $CaSO_4$.

2a)

In 1987, farmers 1 and 3 could pump with a radius of influence of 1/4th of a mile.

r_o = 0.25 * 1609 = 402 m

Effective pumping happens when h_w is approximately 1/3rd of h_o.

h_o = 50 – 2 = 48 m
h_w = 1/3 * 48 = 16 m

$$Q_1 = pK\frac{(h_o^2 - h_w^2)}{\ln\left(\frac{r_o}{r_w}\right)} = 3.14 * 5 * \frac{(48^2 - 16^2)}{\ln(402/0.1)} = 1181\frac{m^3}{day} ==> 217\ gpm$$

$$Q_1 = pK\frac{(h_o^2 - h_w^2)}{\ln\left(\frac{r_o}{r_w}\right)} = 3.14 * 5 * \frac{(48^2 - 16^2)}{\ln(402/0.05)} = 1089\frac{m^3}{day} ==> 200\ gpm$$

2b)

Evaporation is then total volume for 1 day over the acreage (160 acres = 16.2 ha).

For field 1: $1181/16.2*10^4$ = 7.3 mm
For field 2: $1089/16.2*10^4$ = 6.7 mm

Average evaporation rate was approximately 7 mm/day

2c)

This time, the situation is WITH recharge, and the radius of influence changes to 1/8th of a mile.

$$Q_1 = pK\frac{(h_o^2 - h_w^2)}{\ln\left(\frac{r_o}{r_w}\right) - \frac{1}{2}} = 3.14 * 5 * \frac{(48^2 - 16^2)}{\ln(201/0.1)} = 1379\frac{m^3}{day} ==> 253\ gpm$$

2d) The equations given in the Grismer book are only valid for when the recharge rate equals the pumping rate. The derivation shown in the handout needs to be adjusted for a case where w π r^2 = 0.1*Q. Thus: No, the equations as given can not be used in this case.

3a)

Although water is pumped from a depth of 30 m, the actual height that water has to be lifted is 15 m – 5 m = 10 m.

The sand filter, with a high hydraulic conductivity, creates a head loss that can be calculated using Darcy

$$\frac{Q}{A} = -K\frac{dH}{dL}$$

In a pure horizontal flow, we can simplify this as

$$\frac{Q}{A} = -K\frac{dh}{dL}$$

solving for Δh gives

$$\frac{1200/24}{\pi\, 0.4^2} = -0.05\frac{dh}{1}$$

$$-\frac{1200/24}{0.05\pi\, 0.4^2} = dh = -1989m$$

Negative sign indicates that this is a head loss.

Total pump head is then the head loss + the elevation head + velocity head = 1989 + 10 + 0.001 = 1999 m

Calculating the power requirement for the pump:

$$H_P = \frac{P}{\gamma Q} = 1999 = \frac{P}{9800 * 0.014}$$

P = 272 kW

However, the pump is only 75% efficient, thus the real pump size we need to install for this system is:

272/0.75 = 363 kW

3b)

Critical depth can be calculated from the equation:

$$\frac{Q^2}{g} = \frac{A^3}{B}$$

Where A = cross sectional area
And B = width at the water surface

Set y = critical depth

B = 0.2 through the weir
A = 0.2 * y
Q = 600 m^3/day = 6.94 * 10^{-3} m^3/s

$$\frac{(6.94 * 10^{-3})^2}{9.81} = \frac{(0.2y)^3}{0.2}$$

y = 5.0 cm

3c)

Using the Manning equation:

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}}$$

R = A/wp = (0.2 * 0.05)/(0.2+2*0.05) = 0.033

6.94 * 10^{-3} = (1/0.012) * (0.2 * 0.05) * $(0.033)^{0.67}$ * $S^{0.5}$

S = 6.58 * 10^{-3} → 6 mm per meter → 6.6 m per kilometer