2. (a) liquid B because it is a shorter column than A

(b) pressures - at pt. ①: $p = 0$ (atmospheric)

pt. ②: $p = \rho_{Hg} g (h_1 - h_2) = \rho_w g (h_3 - h_2)$

pt. ③: $p = 0$

4. Continuity and Bernoulli Eq. application

a) $V_1 = 2\ m/s$ $D_1 = 8\ cm \Rightarrow A_1 = \pi (0.04\ m)^2 = 0.0016\pi\ m^2$

$V_2 = ?$ $D_2 = 3\ cm \Rightarrow A_2 = \pi (0.015\ m)^2 = 0.000225\pi\ m^2$

$V_3 = ?$ $D_3 = 6\ cm \Rightarrow A_3 = 0.0009\pi\ m^2$

$$Q = V_1 A_1 = (2\ m/s)(0.0016\pi\ m^2) = 0.01\ m^3/s$$

$$V_2 = \frac{0.01\ m^3/s}{0.000225\pi\ m^2} = \underline{\underline{14.3\ m/s}}$$

$$V_3 = \frac{0.01}{0.0009\pi} = \underline{\underline{3.57\ m/s}}$$

b) taking $z = 0$ at the pipe centerline and noting that 1 psi $\simeq$ 70 cm of H_2O

$$H_1 = \frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} = 3(70\ cm\ H_2O) + 0 + \frac{4\ m^2/s^2}{2(9.8)}$$

$$= 2.1\ m + 0.2\ m = 2.3\ m$$

$$H_2 = H_1 - H_L = 2.3\ m - 0.5\ m = 1.8\ m$$

$$= \frac{p_3}{\gamma} + \frac{V_3^2}{2g} \quad \text{or} \quad \frac{p_3}{\gamma} = 1.8\ m - \frac{(3.57)^2}{2(9.8)} = 1.16\ m$$

$$= \underline{\underline{116\ cm\ of\ H_2O}}$$

6. The driving force behind the flow is the head loss between the two reservoirs, that is, $h_f = 80 - 65 = 15\ m$. Assuming turbulent flow

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 15\ m$$

$$V = \sqrt{\frac{(0.12\ m)(2)(9.81)(15)}{(0.025)(460)}} = 1.75\ m/s$$

$$Q = VA = 1.75\pi(0.06\ m)^2 = 0.020\ m^3/s = \underline{\underline{20\ l/sec}}$$

8. $H_1 + H_p = H_2$; $Q = 28\ \ell/\text{sec} = 0.028\ m^3/s$

$$h_1 + \frac{v_1^2}{2g} + H_p = h_2 + \frac{v_2^2}{2g}$$

$h_1 \downarrow$ 30 m $h_2 \downarrow$ 60 m

The manometers indicate a piezometric pressure head because they are referenced to some elevation datum not specified in the problem.

$D_1 = 0.15\ m$, $D_2 = 0.10\ m$

$$V_1 = \frac{0.028\ m^3/s}{\pi(0.075\ m)^2} = 1.58\ m/sec$$

$$V_2 = \frac{0.028\ m^3/s}{\pi(0.05)^2} = 3.565\ m/s$$

$$H_p = (60-30) + \frac{1}{2g}(v_2^2 - v_1^2) = 30\ m + \frac{1}{2g}(10.2) = 30.52\ m$$

$$P = H_p Q \gamma = (30.52\ m)(0.028\ \tfrac{m^3}{s})(9800\ \tfrac{N}{m^3})$$
$$= \underline{\underline{8.375\ kW}}$$

10 Determine the flowrate if $P = 40\ kW$ rather than 15.55 as above

Q	f(Q)
0.1	0.968
0.11	1.204
0.105	1.081
0.102	1.012

$$H_p = H_2 - H_1 = (z_2 - z_1) + \left(\frac{p_2}{\gamma} - \frac{p_1}{\gamma}\right) + \frac{1}{2g}(v_2^2 - v_1^2) \ ; \quad V = \frac{Q}{A}$$

$$= (21-0) + (0-6) + \frac{Q^2}{2g}\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = 15 + Q^2\,\frac{48034}{2g} = H_p$$

$$P = 40\,000\ W = \left[15 + 2450.7\,Q^2\right] Q\gamma \ \text{or} \ 15Q + 2450.7\,Q^3 = \frac{40\,000}{9800}$$

$$2450.7\,Q^3 + 15Q = 4.08 \quad \text{trial \& error solution for } Q$$

$$f(Q) = 600.66\,Q^3 + 3.676\,Q = 1 \ ; \quad Q \simeq \underline{\underline{0.102\ m^3/sec}}$$

Increasing the pump energy did little for increasing the flowrate!

12 Basically, this question is asking for the upstream pressure head.

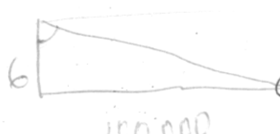
$Q = 0.085\ \frac{m^3}{s}$
$D_1 = 0.3\ m$
$D_2 = 0.1\ m$
$V_1 = 1.2\ m/s$
$V_2 = 10.82\ m/s$

$$\left.\begin{aligned} H_1 &= h + 0 + \frac{V_1^2}{2g} \\ H_2 &= 0 + 0 + \frac{V_2^2}{2g} \end{aligned}\right\} \quad H_1 = H_2 + H_{turbine} = H_2 + \underbrace{\frac{15\,000}{(0.085)(9800)}}_{18\ m}$$

$$h = 18\ m + \frac{1}{2g}(v_2^2 - v_1^2) = 18 + \frac{1}{2g}(115.7) = \underline{\underline{23.9\ m}}$$

14 The slope of the pipe is the same as the head loss per meter, or h_f/L.
Assuming turbulent flow again.

$$\frac{h_f}{L} = \frac{f}{D}\frac{v^2}{2g} \qquad \text{and } v = \frac{Q}{A} = \frac{200\ \ell/s}{\pi(0.5\ m)^2}\cdot\frac{m^3}{1000\ \ell} = 0.255\ m/s$$



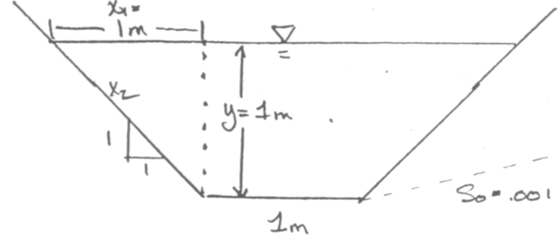
$f \simeq 0.018$ (Moody Diagram)

$$\frac{\epsilon}{D} = \frac{0.36\ mm}{1000\ mm} = 0.00036$$

$$\mathbb{R} = \frac{vD}{\nu} = \frac{(0.255\ \frac{m}{s})(1\ m)}{1.31\times10^{-6}\ \frac{m^2}{s}} = 194\,388 \simeq 2\times10^5$$

$$\frac{h_f}{L} = \text{slope} = \frac{0.018}{1}\cdot\frac{(0.255)^2}{2(9.8)} = 0.00006 \ \text{or} \ \underline{\underline{0.006\ \%}}$$

1 / 100,000



18 $Q = ?$

$n = 0.024$

$Q = AC\sqrt{RS_o}$ $\quad C = R^{1/6}/n$ $\quad R = \frac{A}{W_p}$

① Area: $1m^2 + \frac{1}{2}(1m)(1m) \times 2$ $\quad$ * we know the ht. (y) b/c the side slopes are 1:1, we know x_1

$= 2m^2$

② $W_p = 1m + 2(1.4m^*)$ $\quad x_2 = 1/\sin 45°$

$= 3.8m$

③ R (hydraulic radius) $= \frac{2m^2}{3.8m} = \cancel{.71m}$ ~~0.526~~ $\boxed{0.5224} = R$

④ $C = \cancel{.71m^{1/6}}/.024 = \cancel{39.4}$ 37.4 $\quad C = \frac{0.5224^{1/6}}{0.024} = \frac{.8974}{0.024} = \boxed{37.3}$ C

So, $Q = (2m^2)(\underbrace{37.4)(.567)^{1/2}}_{m}(.001)^{1/2} =$ 1.72 m^3/s $\quad Q = (2m^2)(37.4)[(0.5224)(.001)]$

note that units don't work out —

$= \boxed{\cancel{6.64 m^3/s}}$

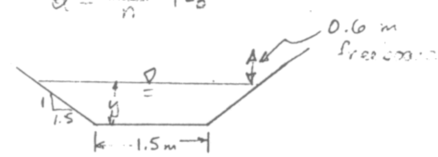
$\boxed{Q = 1.709 \, m^3/s}$

20. Open-Channel problem $Q = AC\sqrt{RS_o}$ and $C = \frac{R^{1/6}}{n}$

$Q = 200 \, l/sec = 0.2 \, m^3/sec$ $\quad Q = \frac{AR^{2/3}}{n}\sqrt{S_o}$

$S = 0.001$

$n = 0.04$



$A = \frac{1}{2}[\overset{b_1}{1.5m} + \overset{b_2}{(1.5 + 2(1.5y)}]y$

$wp = 1.5 + 2((1.5y)^2 + y^2)^{1/2} = 1.5 + \sqrt{11.5}\,y$ $\quad ((1.5)^2y^2 + y^2)^{\frac{1}{2}} * 2$

$= 1.5y + 1.5y^2$

$R = \frac{A}{wp} = \frac{1.5(y+y^2)}{1.5 + \sqrt{11.5}\,y}$

$= \frac{y + y^2}{1 + 0.82y}$

$AR^{2/3} = Qn/\sqrt{S_o} = (0.2)(0.04)/(0.001)^{1/2} = 0.253$

$f(y) = 1.5(y+y^2)\left[\frac{y+y^2}{1+0.8y}\right]^{2/3} = 0.253$ $\quad$ requires a trial & error solution

$3.0\left(\frac{2}{1.8}\right)^{2/3}$

y	f(y)
1	3.2
0.3	0.27
0.4	0.47
0.28	0.236

y = 0.29 m

(a) water depth = y = <u>29 cm</u>

(b) Top width = 1.5 + 3(.89) = <u>4.17 m</u>

(c) water surface width = <u>2.37 m</u>

22 ~~#~~. Force balance problem similar to class notes – convert to SI units for ease

$$F + (P_1A_1 - P_2A_2) = \rho Q(V_2 - V_1) \quad \text{where } D_1 = 1 \text{ in} \doteq 0.0254 \text{ m}$$
$$D_2 = \tfrac{1}{4} \text{ in} = 0.0063 \text{ m}$$

$$Q = 20 \frac{gal}{min} \cdot \frac{m^3}{264.2\, gal} = 0.073\ m^3/min$$

Absolute pressures are necessary ($1\ atm = 14.7\ psi = 1030\ cm\ of\ H_2O$)

$$P_1 = 45 + 14.7\ psi = 59.7\ \cancel{psi} \cdot \frac{0.704\ m}{\cancel{psi}} \cdot \frac{9800\ N}{m^3} = 411.9\ kPa$$

$$P_2 = 14.7\ psi = 14.7(0.704)(9800) = 101.4\ kPa$$

$$(P_1A_1 - P_2A_2) = (411.9\ kPa)(0.0254\ m)^2 \cdot \frac{\pi}{4} - (101.4)(0.0063)^2 \cdot \frac{\pi}{4}$$
$$= 208.7\ N - 3.16\ N$$
$$= 205.5\ N$$

$$\rho Q(V_2 - V_1) = \left(1000 \frac{kg}{\cancel{m^3}}\right)\left(0.073 \frac{\cancel{m^3}}{\cancel{min}}\right)\left(\frac{\cancel{min}}{60\, s}\right)(\underbrace{V_2 - V_1}_{36.6}) = 44.5\ N$$

$$V_1 = \frac{Q}{A_1} = \frac{0.073\ m^3/\cancel{min}}{\frac{\pi}{4}(0.0254\ m)^2} \cdot \frac{\cancel{min}}{60\ sec} = 2.4\ m/s$$

$$V_2 = \frac{0.073}{\frac{\pi}{4}(0.0063)^2} \cdot \frac{1}{60} = 39.0\ m/s$$

$$(V_2 - V_1) = 36.6\ \frac{m}{s}$$

$$\therefore \quad F = 44.5\ N - 205.5\ N = \underline{\underline{-161\ N}}$$ (the minus sign indicates $\xleftarrow{F}$)